CS256-Assignment#3

3 Problems. 50 points.

Finish reading Chapter 0 of the textbook (excluding Section 0.4 and the parts related to grouped statements).

1. [30 points] Prove that every formula of the form

$$\nabla_1 \nabla_2 \ldots \nabla_n p,$$

where each ∇_i is either $\Box$ or $\Diamond$, is congruent to one among

$$\Box p, \quad \Diamond p, \quad \Box \Diamond p, \quad \textit{and} \quad \Diamond \Box p.$$

For example, $\Diamond \Box \Box \Diamond \Diamond \Diamond p \approx \Box \Diamond p$.

Describe a procedure for computing the correct congruence.

<u>Note:</u> Prove the congruences, such as $\Box \Box p \approx \Box p$, used in your proofs.

2. [10 points] Prove that: $\widehat{\mathcal{W}}$, $\wedge$, $\neg$, $\forall$
is a complete set of operators for future temporal formulas, i.e., every future temporal formula is congruent to a formula built from state formulas using only $\widehat{\mathcal{W}}$, $\wedge$, $\neg$, and $\forall$.
Clarification: $\forall$ is only included because temporal logic formulas include the quantifiers $\forall$ and $\exists$. Don't use $\forall$ when showing how to rewrite a temporal operator (e.g., by just opening up its definition). [Give the temporal congruences without proof.]

3. [10 points]
(a) Write a quantifier-free formula stating that p holds precisely at all even positions, i.e., p is true at positions 0,2,4,... and false at positions 1,3,5,....

(b) Using flexible quantification over boolean variables, write a formula stating that p holds at all even positions. The formula should not restrict the value of p at odd positions.

(c) Using flexible quantification over integer variables, write a formula stating that p holds at positions 0,1,4,9,16,..., i.e., positions $j = k^2$ for $k = 0, 1, \ldots$. Nothing is said about the value of p at other positions. You can also use rigid quantification if necessary.